

## Math 1010 November 15, 2006 Homework 24

Text problems: 7.4 # 1, 3

### Partial Fractions

We want to decompose a fraction into a sum of simpler fractions. There are several cases, but here we concentrate on one case where the factors of the denominator are distinct and linear.

Example: Decompose  $\frac{x+6}{x^2+3x+2}$ . The denominator factors:  $3x + 2 = (x + 1)(x + 2)$ .

We want to find numbers A and B so that  $\frac{x+6}{(x+1)(x+2)} = \frac{A}{x+2} + \frac{B}{x+1}$ .

First, multiply both sides by the common denominator to clear denominators.

$$\frac{(x+6)(x+1)(x+2)}{(x+1)(x+2)} = \frac{A(x+1)(x+2)}{x+2} + \frac{B(x+1)(x+2)}{x+1}; \text{ Clearing, we get } x + 6 = A(x + 1) + B(x + 2),$$

and multiplying out and collecting we get  $x + 6 = Ax + Bx + A + 2B$

Separating the x-terms and numbers into two equations, we get

$$\left\{ \begin{array}{l} x = Ax + Bx \\ 6 = A + 2B \end{array} \right\} \text{ which leads to the system } \left\{ \begin{array}{l} 1 = A + B \\ 6 = A + 2B \end{array} \right\} \text{ or } \left\{ \begin{array}{l} A + B = 1 \\ A + 2B = 6 \end{array} \right\}$$

Solve this system using any method, say substitution: Solution is:  $[A = -4, B = 5]$

Thus, for the equation  $\frac{x+6}{(x+1)(x+2)} = \frac{A}{x+2} + \frac{B}{x+1}$  we have found values for A and B making it true, so

$$\frac{x+6}{(x+1)(x+2)} = \frac{-4}{x+2} + \frac{5}{x+1}.$$

1) Decompose the fraction  $\frac{2x+1}{x^2-7x+12}$